Fluid Mechanics:-

Fluid mechanics is the study of fluids (includes liquid, gases and plasmas) and the forces on them.

Stream line and Turbulent flow:-

When a liquid flows steadily, such that each particle passing a certain point flows exactly the same path and has the same velocity as its preceding particle, the flow is said stream line.

When the velocity of liquid reaches the critical velocity takes a zig-zag path. This state shows that motion is no longer steady. This is called turbulent flow.

Viscosity:-

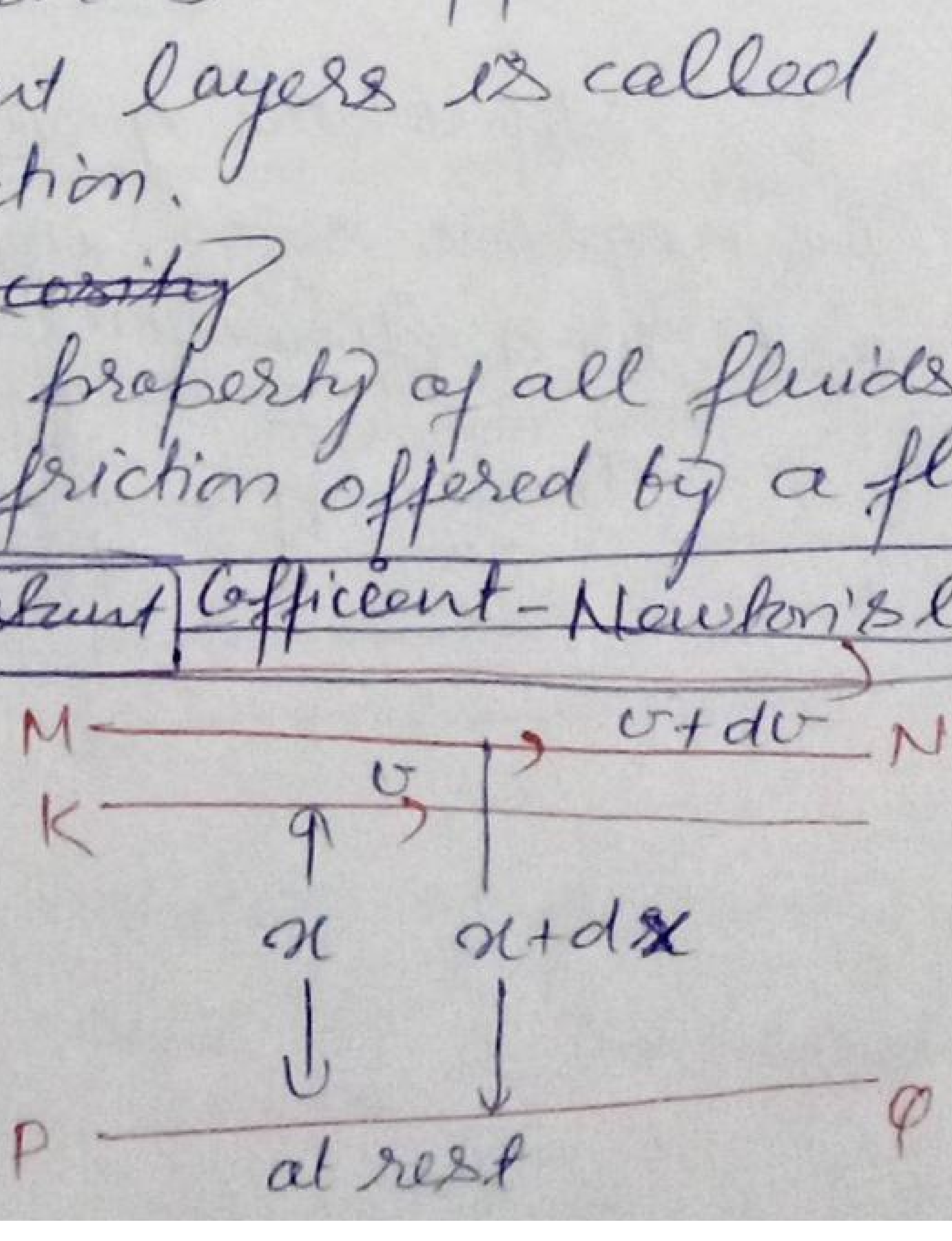
This tangential backward dragging force, coming into play in between two adjacent layers of liquid and tending to oppose the relative motion between them is called the viscous force or viscous drag.

The property of liquid which rise to such viscous forces or which tends to oppose relative motion between its different layers is called viscosity or internal friction.

or (~~Viscosity~~

Viscosity is the inherent property of all fluids and may be called the internal friction offered by a fluid to its flow. | Viscosity ~~Constant~~ Cofficient - Newton's law

Consider a liquid flowing in a stram line over a fixed horizontal surface PQ.



The layer immediately in contact with the fixed surface will be at rest while the velocity of other layers increases uniformly with their distance from this surface.

The property by virtue of which a liquid resists the relative motion between its different layers is called viscosity.

Now according to Newton's law of viscous flow for stream line motion, the tangential viscous drag F acting between two layers of area A, at a distance dx apart moving with relative velocity dv is directly proportional to

(i) area A of the layers.

(ii) the rate of change of velocity with distance i.e. velocity gradient dv/dx.

$$F \propto A \frac{dv}{dx}$$

$$\boxed{F = -\eta A \frac{dv}{dx}}$$

where η is coefficient of viscosity.

The negative sign indicates that the viscous force acts in a direction opposite to the flow of liquid.

Thus $$\eta = \frac{F}{A \cdot \frac{dv}{dx}} = \frac{[MLT^{-2}]}{[L^2]\left[\frac{LT^{-1}}{L}\right]} = [ML^{-1}T^{-1}]$$

Flow of liquid through a capillary: Poiseuille's Formula:-

Consider a cylindrical horizontal capillary tube of radius 'a' and length 'l' through which a liquid is flowing under a constant pressure difference p applied between the ends of the tube.

It is assumed that

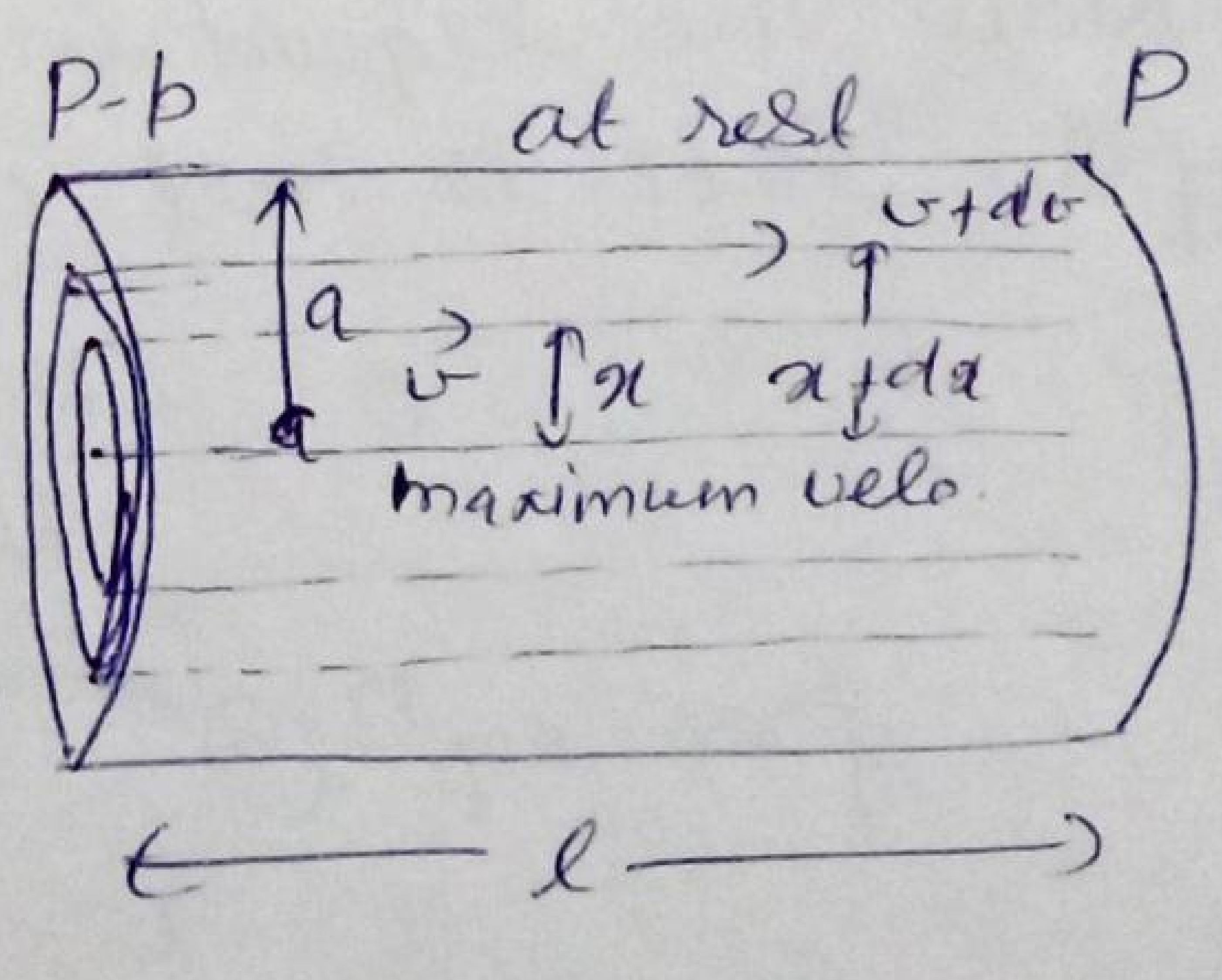
(i) The flow of liquid is stream lined and everywhere parallel to the axis of the tube.

(ii) The flow is steady i.e there is no acceleration of the liquid at any point.

(iii) The pressure will be constant over any cross-section

(iv) The velocity of the layer of liquid in contact with the walls of the tube is zero and increases regularly and continuously towards the interior becoming maximum along the axis of the tube.

Newton's law of viscous flow



$$F = -\eta A \frac{dv}{dx}$$

$$F = -\eta \cdot 2\pi x l \frac{dv}{dx} \quad \text{--- (1)}$$

where η is the coff. of viscosity of liquid, v is the velocity of flow at a distance x from the axis and $\frac{dv}{dx}$ is the velocity gradient.

Now the flow of liquid is maintained by a forward force due to the difference of pressure p between the ends of the tube.

Hence the force tending to accelerate the cylindrical liquid layer.

$$= \text{Pressure difference} \times \text{area of cross-section}$$

$$= p \times \pi x^2 \quad \text{—— (2)}$$

equating (1) and (2), we get

$$-\eta \cdot 2\pi x l \frac{dv}{dx} = \pi x^2 \times p$$

$$dv = -\frac{p}{2\eta l} x\, dx$$

integrating this expression, we get

$$v = -\frac{p}{2\eta l}\frac{x^2}{2} + C \quad \text{—— (3)}$$

where C is constant.

Now the liquid in contact with the walls of the tube is rest i.e $v = 0$ when $x = a$

$$0 = -\frac{p}{2\eta l}\frac{a^2}{2} + C \Rightarrow \quad C = \frac{pa^2}{4\eta l}$$

from eqn (3)

$$v = -\frac{px^2}{4\eta l} + \frac{pa^2}{4\eta l}$$

$$\boxed{v = \frac{p}{4\eta l}(a^2 - x^2)}$$

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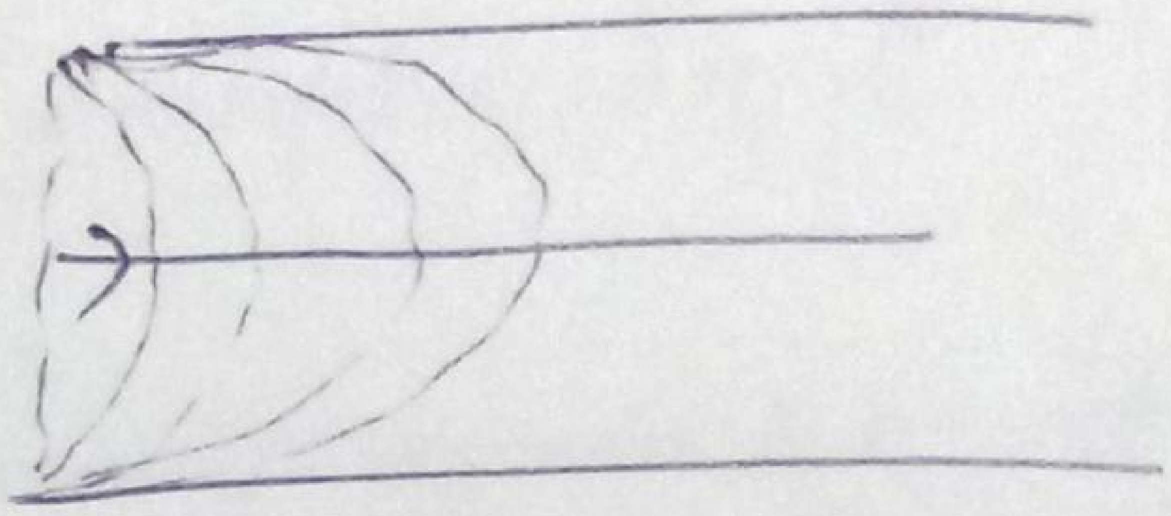
This gives the velocity of flow at a distace x from the axis of the tube and shows that the profile of ~~do~~ advancing liquid is a parabola



Energy possessed by a flowing liquid:-

A liquid in flow possesses three forms of energies:

① **Pressure Energy**:- If a pressure p is exerted on an area 'a' of the liquid and liquid moves through a displacement x under this pressure, then the work done in the process is given by

$$W = \text{force} \times \text{displacement}$$
$$= pa \cdot x$$

~~The potential energy of a~~ liquid ~~of mass m at a height~~

The work done on the mass $ax \cdot \rho$ of the liquid resides in it in the form of its pressure energy hence

$\left[\because \text{density} = \frac{\text{mass}}{\text{vol.}} \right]$

(ii) **Potential energy**:

A liquid possesses potential energy due to its position. The potential energy of a liquid of mass m at a height h above earth's surface is equal to mgh. Hence

Potential energy per unit mass of the liquid $= \frac{mgh}{m} = gh$

(iii) **Kinetic energy**:-

~~Scien~~ Since a liquid has inertia, it possesses kinetic energy. The kinetic energy of a mass m of a liquid flowing with a velocity v is given by $\frac{1}{2}mv^2$, so that

Kinetic energy per unit mass of the liquid $= \frac{1}{2}v^2$

~~The Kinetic~~ The three forms of energy possessed by a liquid under flow are mutually convertible.

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Bernoulli's Theorem:-

It is the fundamental theorem of hydrodynamics and expresses the variation of pressure with the velocity of flow along a stream-line. It states that the total energy (pressure energy + kinetic energy + potential energy) of an incompressible nonviscous fluid ~~flow~~ flowing from one point to another, without any friction, remains constant throughout the displacement. We ~~have~~ have all energy

$$\boxed{p + \frac{1}{2}\rho v^2 + \rho g h = \text{Const}}$$

Proof:- Euler's equation ~~we~~ is

$$\rho\left(v\frac{\partial v}{\partial l} + \frac{\partial v}{\partial t}\right) = -\frac{\partial p}{\partial l} - \rho g\frac{\partial h}{\partial l} \quad (1)$$

for steady or stream line flow

$\frac{\partial v}{\partial t} = 0$, Therefore eqn (1) becomes

$$\rho v\frac{dv}{dl} = -\frac{\partial p}{\partial l} - \rho g\frac{\partial h}{\partial l}$$

This may be expressed as

$$\rho v\,dv + \partial p + \rho g\,\partial h = 0$$

$$v\,dv + \frac{\partial p}{\rho} + g\,\partial h = 0$$

Integrating above eqn., stream line,

incompressible. $\rho = \text{Constant}$

$$\frac{v^2}{2} + \int\frac{dp}{\rho} + gh = \text{const.}$$

$$\boxed{\frac{v^2}{2} + \frac{p}{\rho} + gh = \text{Constant}}$$

This is called Bernoulli's theorem:-

$$p + \frac{1}{2}\rho v^2 + \rho g h = \text{constant}$$

$$\frac{p}{\rho g} + \frac{v^2}{2g} + h = \text{Constant}$$

Every term in this equation has dimensions of length and is called a head. This term $\frac{p}{\rho g}$ is called the pressure head, term $\frac{v^2}{2g}$ is called velocity head, and the term h is called the potential head or gravitational head.

For a fluid moving horizontally $h_1 = h_2 = \text{Const.}$ 2
and Bernoulli's theorem reduces to the form.

$$\frac{p}{\rho g} + \frac{v^2}{2g} = \text{Constant}$$

$$\boxed{p + \frac{1}{2}\rho v^2 = \text{Constant}}$$

This expression shows that pressure and velocity can only increase at the expense of one another. i.e points of maximum pressure correspond to those of maximum velocity and vice versa. The priciple is used ~~is~~ in the design of flow measuring instrument e.g. ~~ventu~~ venturimeter, picot tube and in the constructing of various exhaust jet pumps.