B.Sc Part I

UNIT-I

PHYSICS

PAPER I: MECHANICS

Frame of Reference :-

A system of coordinate axis, relative to which the motion of an object is described, is called a frame of reference. This a frame of a reference is a a system of coordinate axis which defines the position of a particle or an event in two or three dimensional space.

The simplest frame of reference is the cartesian co-ordinate system. Then the position vector of the particle with respect to the origin is flloweed by

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

where $\hat{i}, \hat{j}, \hat{k}$ are the unit vectors along the three axes.

velocity of a particle is given by

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$$

and its acceleration by

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$$

Note :- The origin of the frame of reference may not always be coincident with the position of the observer.

For complete, identification of an event, we require its position and time of occurrence. Therefore, in addition to the usual three spatial coordinate x, y and z we need another co-ordinate that of time t. A reference frame with such four co-ordinate (x, y, z, t) is called space-time frame of reference. Ex: Our earth, Walls of the room.

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Inertial frames of Reference:-

Those frames of references, in which Newton's first and second law hold true are called inertial frames. They are also known as Newtonian or Galilean frames.

Thus, inertial frames are unaccelerated. If a frame is accelerated, a body moving with constant velocity will appear accelerated in this frame.

Non-Inertial frames:

~~The~~ However, a frame of reference is accelerated relative to an inertial frame, the form of the fundamental physical laws, such as Newton's second law, become become completely different. Such accelerated frames of reference are called non-inertial frames. Therefore, non-inertial frames may be defined as those frames in which Newton's first law and second law are not valid. All accelerated and rotating frames are non-inertial frame of references.

Newton's Laws of Motion:-

First law:- A body must continue in its state of rest or of uniform motion along a straight line unless acted upon by an external force.

Second law:- The rate of change of momentum is proportional to the impressed force and takes place in the direction of the force.

Third law:- To very action, there is an equal and opposite reaction. Thus if $\vec{F}_{12}$ and $\vec{F}_{21}$ be the force exerted on each other by two interacting bodies. We have $\vec{F}_{12} = -\vec{F}_{21}$

Limitations of Newton's Laws of Motion:-

First Limitation:- The relation $\vec{F} = m\vec{a}$ would not good in case m does not remain constant.

Ex:- (i) A falling raindrop which gathers mass as it falls
(ii) A rocket, which continuously losses part of its mass
(iii) Particles with relativistic velocities

Second limitation:- Both the forces are measured simultaneously.

Dynamics:-

According to Einstein, "The purpose of mechanics is to describe how bodies change their position in space with time". Dynamics is the study of Motion of bodies and relationship of this motion with the force producing it

(i) Dynamics of uncharged Particle

(ii) Dynamics of Charged Particle

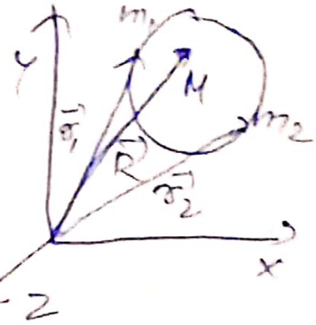
Centre of Mass : (C.M.)

A system consisting of a large number of particles. There is one point in it which behaves as though the entire mass of the system were concentrated there and its motion is the same as would ensue if the external forces acting on the system were applied directly to it. This point is called the centre of mass of the system.

Let us consider a system has two particles of masses m_1 and m_2 and total mass M, with $\vec{r}_1$ and $\vec{r}_2$ as their position vectors with respect to origin. If $\vec{R}$ be the position vector of the centre of mass, we have

$$\vec{R} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1+m_2} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{M}$$

$$M\vec{R} = m_1\vec{r}_1 + m_2\vec{r}_2$$



i.e the product of total mass of the system and the position vector of the centre of mass is equal to the sum of the products of the individual masses and their respective position vectors.

Motion of velocity of C.M :-

Let us consider the system consisting of n particles and total mass M assuming that the mass of the system remains constant.

$$M\vec{R} = \Sigma m_i\vec{r}_i = m\vec{r}_1 + m_2\vec{r}_2 + \dots + m_i\vec{r}_i + \dots m_n\vec{r}_n$$

Differentiating w.r.t to time

$$M\frac{d\vec{R}}{dt} = m_1\frac{d\vec{r}_1}{dt} + m_2\frac{d\vec{r}_2}{dt} + \dots \frac{m_i d\vec{r}_i}{dt} + m_n\frac{d\vec{r}_n}{dt}$$

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$$\frac{d\vec{R}}{dt} = \vec{V} \text{ the velocity of centre of mass}$$

$$\frac{d\vec{r}_1}{dt} = \vec{v}_1 , \frac{d\vec{r}_2}{dt} = \vec{v}_2$$

$$M\vec{V} = m_1\vec{v}_1 + m_2\vec{v}_2 + \dots + m_r\vec{v}_r + \dots + m_n\vec{v}_n = \sum m_i\vec{v}_i \quad (1)$$

The velocity of centre of mass is

$$\vec{V} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + \dots + m_r\vec{v}_r + \dots + m_n\vec{v}_n}{M} = \frac{\sum m_i\vec{v}_i}{M} \quad (2)$$

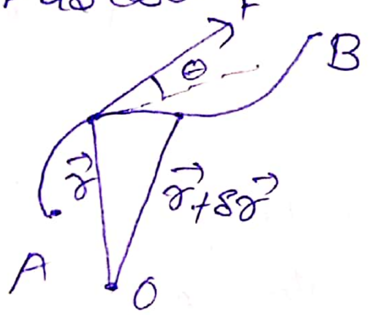
From eq (1), we also find the vector sum of the linear momenta of the individual particles i.e the total linear momentum of the system required to the product of the total mass of the system and the velocity of the centre of mass

Law of Conservation:-

Work:- A Work done by the force, $dW = \vec{F}\cdot\vec{ds}$

Total work done by the force in displacing the particle along the whole length of the curve from A to B

$$W = \int_A^B \vec{F}\cdot\vec{dr} = \int_A^B \vec{F}\cdot\vec{ds} = \int_A^B F\,ds\cos\theta$$



Conservation of force:-

The workdone by a conservative force in displacing a particle from one point to another depends only on the position of the two given points, a ~~fixed centre~~ and independent of the actual path taken between them. On the other hand, the work done by a non-conservative force does depend upon the actual path taken.

Conservative Ex- Central force such as gravitational, Electrostatic. (Position dependent)

Non-Conservative → friction, viscous force (velocity dependent)

Energy:- The energy of a particle is its capacity to do work

(i) **Kinetic Energy-Work Energy principle**:- Kinetic energy (T) of a particle is due to its motion.

Work done against applied force

$$W = -\int \vec{F}\cdot\vec{dr}$$

Newton's second law of motion

$$\vec{F} = m\frac{d\vec{v}}{dt}$$

$$W = -m\int \frac{d\vec{v}}{dt}\cdot d\vec{r} = -m\int \frac{dr}{dt}\, dv = -m\int v\, dv$$
$$= \frac{1}{2}mv^2$$

Potential Energy:- The potential energy of a particle as its capacity to do work in virtue of its position.

$$U = \int_{r}^{r_0} \vec{F}\cdot d\vec{r}$$

$$U = \int_{r}^{\infty} \vec{F}\cdot d\vec{r} \quad \text{or} \quad U = -\int_{\infty}^{r} \vec{F}\cdot d\vec{r}$$

The potential energy of a particle at a point $\vec{r}$ is given by the amount of work done is moving it from ∞ to that point.

Work energy Principle:

Newton's second law

$$F = m\frac{dv}{dt}$$

multiplying both sides by v or $\frac{dr}{dt}$ and integrating with respect to t between the limits A and B.

$$\int_{A}^{B} m\frac{d\vec{v}}{dt}\cdot \vec{v}\, dt = \int_{A}^{B} \vec{F}\cdot\frac{d\vec{r}}{dt}\, dt$$

$$\frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2 = \int_{A}^{B} F\cdot d\vec{r} = W$$

Work done by a force on a particle is equal to the change in the kinetic energy of the particle. This is called work energy principle.

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Conservation force as negative gradient of potential energy:-

We know that $U = -\int_{\infty}^{r} \vec{F} d\vec{r}$

$\vec{F}$ and $\vec{r}$ in terms of their components along the three coordinate

$$U = -\int_{\infty}^{r} (F_x \hat{i} + F_y \hat{j} + F_z \hat{k})(d_x \hat{i} + d_y \hat{j} + d_z \hat{k})$$

$$U = -\int_{\infty}^{r} (F_x dx + F_y dy + F_z dz)$$

Partial differentiation w.r. to x, y, z, gives

$$F_x = -\frac{\partial U}{\partial x} \quad F_y = -\frac{\partial U}{\partial y} \quad F_z = -\frac{\partial U}{\partial z}$$

$$\vec{F} = F_x \hat{i} + F_y \hat{j} + F_z \hat{k} = -\left(\frac{\partial U}{\partial x}\hat{i} + \frac{\partial U}{\partial y}\hat{j} + \frac{\partial U}{\partial z}\hat{k}\right) = -\text{grad}\, U$$

Thus, a conservative force F is equal to the negative gradient of potential energy U.

Conservation of Linear Momentum:-

The linear momentum of a body is defined as the product of its mass and linear velocity.

$$\vec{p} = m\vec{v}$$

The total linear momentum of a system of particles free from the action of external forces.

Newton's second law of motion, $\vec{F} = \frac{d\vec{p}}{dt}$

The external force is absent, $\vec{F} = 0$

$$\frac{d\vec{p}}{dt} = 0 \quad \text{or} \quad \vec{p} = \text{Constant}$$

Systems of variable mass :- (The Rocket) :-

Principle :- Rocket propulsion is based on the principle of conservation of linear momentum.

Theory :- let M be the mass of the rocket, V is the velocity at any given instant t. α be the rate of change of mass of the rocket. Assuming the forward velocity of the rocket to be along the same line with oppositely directed velocity v of the hot gases of the jet.

velocity of the jet in the laboratory frame $= -v + V = (V - v)$

rate of change of the momentum $= -\alpha(V - v)$

$$= \frac{dM}{dt}(V - v)$$

Force acting on the rocket

$$\frac{d}{dt}(MV) = \frac{dM}{dt}(V - v) \quad \text{--- (i)}$$

$$\cancel{\frac{dM}{dt}V} + M\frac{dV}{dt} = \cancel{\frac{dM}{dt}V} - \frac{dM}{dt}v \quad \text{--- (ii)}$$

The thrust on the rocket at the instant t.

rewrite eq^n (ii)

$$\cancel{Mdt} \quad dV = -\frac{dM}{M}v$$

integrating w.r. to t

$$\int \frac{dV}{dt}dt = -\frac{1}{M}\int \frac{dM}{dt} v\, dt$$

$$V = -\log M + C$$

if M_0 be the mass of the rocket and V_0 its velocity at $t=0$

$$V_0 = -v\log M_0 + C \Rightarrow C = V_0 + v\log M_0$$

$$V = -\log M + V_0 + v\log M_0$$

$$V = V_0 + v\log\frac{M_0}{M} \longrightarrow \text{(iii)}$$

This gives the value of the velocity V of the rocket at instant t in terms of the mass of the rocket initially and after time t.

Since mass decreases at the rate α

$$M = M_0 - \alpha t$$

$$M = M_0\left(1 - \frac{\alpha}{M_0}t\right) \Rightarrow M = M_0(1-\beta t)$$

where α/M_0 the rate of change of mass in terms of the initial mass M_0.

Substituting this value of M in eq^n (iii)

$$V = V_0 + v\log\frac{M_0}{M_0(1-\beta t)}$$

$$V = V_0 + v\log\left(\frac{1}{(1-\beta t)}\right)$$

$$\boxed{V = V_0 - v\log(1-\beta t)} \longrightarrow \text{(IV)}$$

which gives the velocity of the rocket at the instant t in terms of β.

In case the weight of the rocket is taken into account then

$$\frac{d}{dt}(MV) = \frac{dM}{dt}(V-v) - mg$$

$$\frac{dM}{dt}V + M\frac{dV}{dt} = \frac{dM}{dt}V - \frac{dM}{dt}v - mg$$

$$M\frac{dV}{dt} = -\frac{dM}{dt}v - mg \Rightarrow \frac{dV}{dt} = -\frac{dM}{Mdt}v - g$$

integrating w. r. to t

$$\int \frac{dV}{dt}dt = -v\int \frac{dM}{Mdt}dt - \int g\,dt$$

$$V = -v\log M - gt + C$$

Since $t=0$, $M=M_0$ and $V=V_0$

$$V_0 = -v\log M_0 - g\times 0 + C$$

$$V_0 = -v\log M_0 + C$$

$$\Rightarrow C = V_0 + v\log M_0$$

$$V = -v\log M - gt + V_0 + v\log M_0$$

$$V = V_0 + v\log\frac{M_0}{M} - gt \quad \text{(V)}$$

again $M = M_0(1-\beta t)$

$$\boxed{V = V_0 - v\log(1-\beta t) - gt} \quad \text{(Vi)}$$

(जड़त्वीय)

Notes

$\varphi_1 \rightarrow$ Inertial frame $\rightarrow$ Valid Newton's law, unaccelerated

$\varphi_2 \rightarrow$ Non-inertial frame:- Not valid Newton's law, accelerated
(अजड़त्वीय)

$\Rightarrow$ Conservative force:- Central force $\rightarrow$ gravitational, Electrostatic

$\Rightarrow$ Non conservative force:- friction, viscous force

$\Rightarrow$ Conservative force as negative gradient of potential energy

$$\boxed{\vec{F} = -\text{grad}\, U \text{ or } \vec{F} = -\vec{\nabla} U}$$

$\Rightarrow$ Conservation of Linear momentum:-

$$\vec{F} = \frac{d\vec{p}}{dt} \Rightarrow \vec{F} = 0 \text{ (force should be zero)}$$

$$\frac{d\vec{p}}{dt} = 0 \Rightarrow \boxed{\vec{p} = \text{Constant}}$$

$\Rightarrow$ System of variable mass (rocket):- based on the principle of conservation of linear momentum.

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