Newton's law of Gravitation:-

The law states that every particle of matter in the universe attracts every other particle with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

$$F \propto \frac{mm'}{r^2} \Rightarrow \boxed{F = \frac{mm'}{r^2} G}$$

Gravitational field - intensity of the field:-

The intensity (E) of the gravitational field of a particle of mass m at a point distant r from it is the force experienced by a unit mass placed at that point in the field.

$$\boxed{E = -\frac{m}{r^2} G\hat{r}}$$, where $\vec{r}$ is the unit vector along the direction of $\vec{r}$.

Gravitational potential and gravitational potential energy

The gravitational potential V at a point distant $\vec{r}$ from a body of mass m is equal to the amount of work done in moving a unit mass from infinity (where the gravitational force and potential are zero) to that point.

$$V = -\int_{\infty}^{r} E\,dr = -\int_{\infty}^{r} \cancel{\frac{m}{r^2} G dr - mG}$$

$$V = -\int_{\infty}^{r} \frac{m}{r^2} G\,dr \Rightarrow -mG\left[\frac{1}{r}\right]_{\infty}^{r}$$

$$\Rightarrow \boxed{V = -\frac{m}{r} G}$$

Therefore, that the potential energy of mass 'm'
at that point in question will be

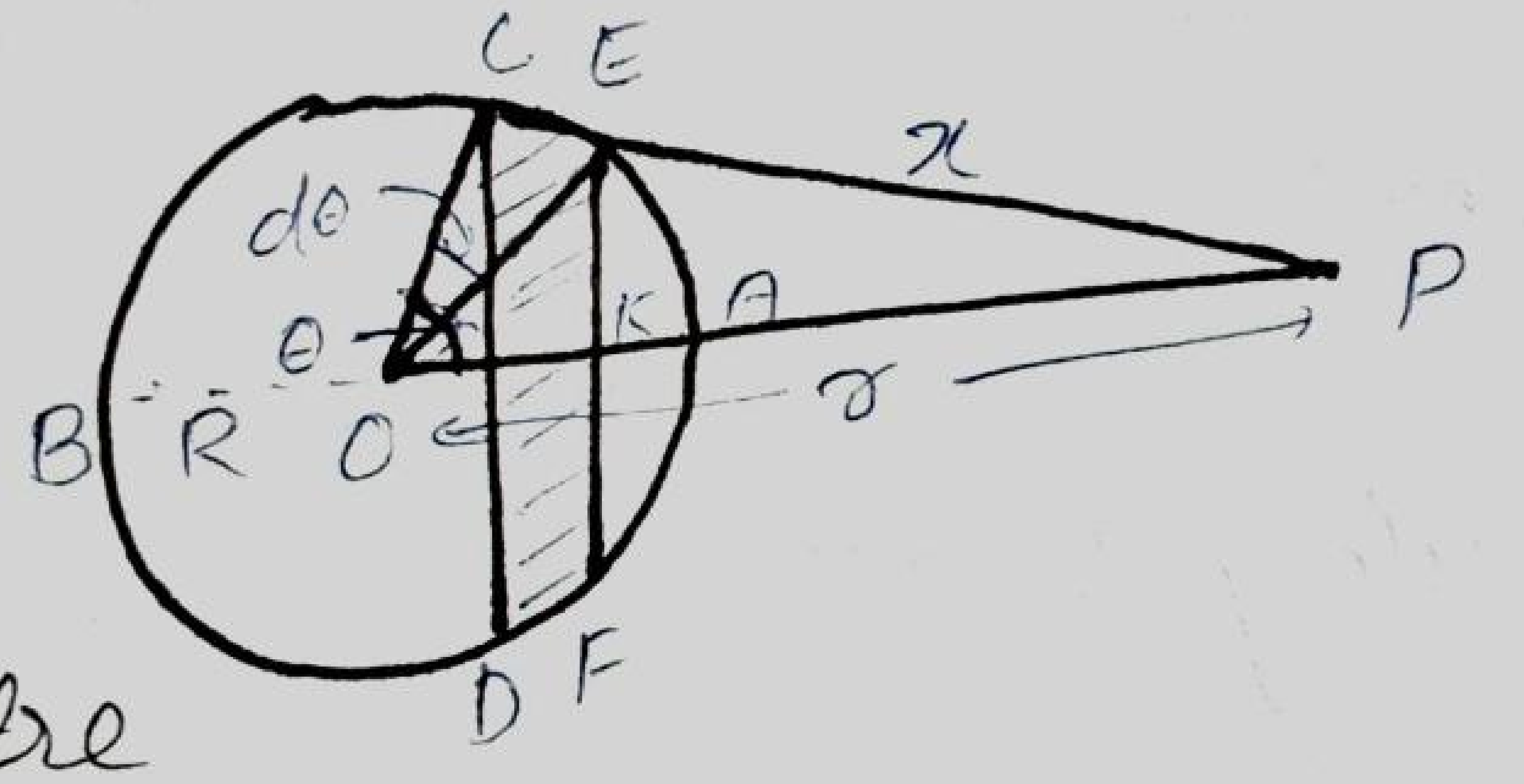
$$\boxed{U = m'V = -\frac{mm'G}{r}}$$

It will be noted that the gravitational ~~energy~~ (V) ~~of a mass 'm' at~~ potential (V) and potential energy (U) are always negative.

Gravitational potential and field due to a spherical shell:-

Gravitational Potential:-

(a) At a point outside the shell:-



let P be a point, distant r from the centre O of a spherical shell of radius R and surface density σ.

Radius of the Ring

$EK = OE\sin\theta \Rightarrow R\sin\theta$ and its circumference $= 2\pi r\sin\theta$, its width $CE = Rd\theta$

Surface area of the ring = circumference x width

$= 2\pi R\sin\theta \cdot Rd\theta$

and its mass $= 2\pi R\sin\theta \cdot Rd\theta\sigma = 2\pi R^2\sin\theta\, d\theta\, \sigma$

Potential at P due to the ring

$$dV = -\frac{\text{mass of slice}}{x} \cdot G$$

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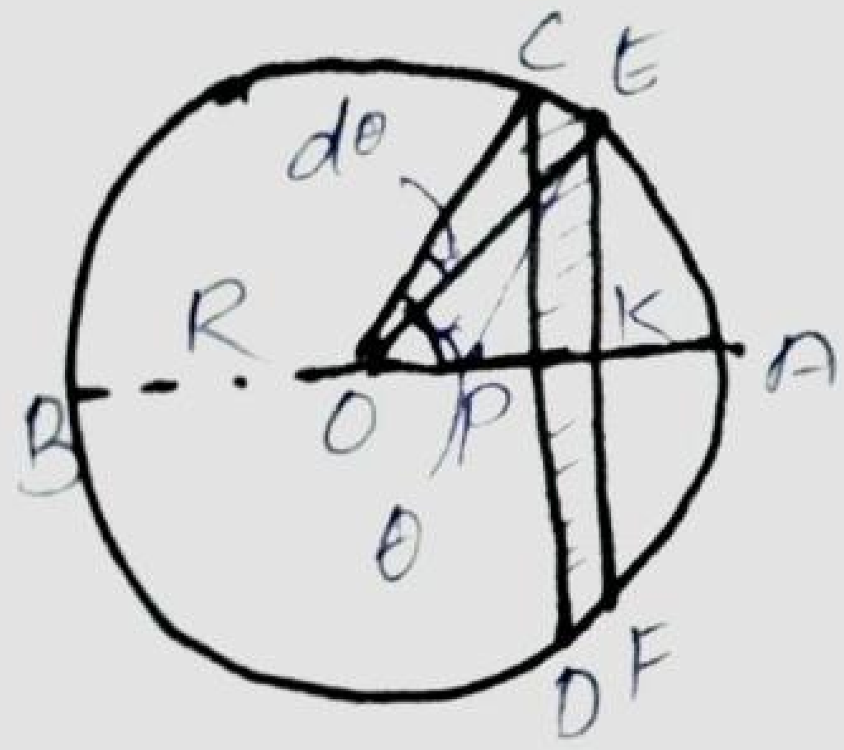
$$= -\frac{4\pi R^2 \sigma G}{r} = -\frac{M}{r}G = -\frac{M}{R}G \quad [\text{here } r = R]$$

Therefore, the mass of the shell behaves as though it were concentrated at its centre

(C) At a point inside the shell :-



Let the point P now lie anywhere inside the spherical shell. Then potential at P due to the ring CEFD

$$dV = -\frac{2\pi R \sigma G}{r}\,dx$$

Potential at P due to the whole shell i.e

$$V = \int_{(R-r)}^{(R+r)} -\frac{2\pi R \sigma G}{r}\,dx$$

$$V = -\frac{2\pi R \sigma G}{r}\left(x\right)_{(R-r)}^{(R+r)} = -\frac{2\pi R \sigma G}{r}\cdot 2r$$

$$V = -4\pi R \sigma G$$

multiplying and diving by R, we have

$$\boxed{V = -\frac{4\pi R^2 \sigma G}{R} = -\frac{M}{R}G}$$

the same as a point on the surface of the shell.

Gravitational field

(a) At a point Outside the shell :-

when $r > R$ the $V = -\frac{Mg}{r}$

Intensity of the gravitational field outside the shell

$$E = \frac{dV}{dr} = -\frac{d}{dr}\left(-\frac{Mg}{r}\right) = \frac{M}{r^2}g$$

or $\boxed{E = -\frac{M}{R^2}g}$ $(\because r = R)$

The same as the whole mass of the shell were concentrated as its centre.

(b) At a point on the surface of the shell :-

$$\because V = -\frac{Mg}{R}$$

$$So \quad E = -\frac{d}{dr}\left(-\frac{Mg}{R}\right) = -\frac{M}{R^2}g$$

(C) At a point inside the shell :-

The gravitational field at a point is given by the space rate of change of potential. So $E = -\frac{dV}{dr}$

Since V is constant for all point inside the shell, So $\frac{dV}{dr} = 0$ i.e the field in the interior of the shell is zero.

Kepler's Laws

(i) **First Law**:- The path of a planet is an elliptical orbit around the sun, with the sun at one of its foci. (law of elliptical orbits)

(ii) **Second Law**:- (law of areas):- The radius ~~of a planet is~~ drawn from the sun to a planet sweeps out equal areas in equal time i.e areal velocity is constant.

(iii) **Third Law**:- The square of a planet's year i.e its time period is proportional to the cube of the semi-major axis of its orbit

$$\cancel{T \propto r} \qquad \boxed{T^2 \propto r^3}$$

Satellite in circular orbit:-

Satellite is a secondary body which revolves around a planet in its own prescribed orbit, planet being a celestial body revolving around the sun.

If M_e be the mass of the earth, m_s the mass of satellite and r be the radius of revolution, then the force of attraction between the earth an satellite.

$$\vec{F} = -\frac{GM_em}{r^2}\hat{r} = -\frac{GM_em}{r^3}\vec{r} \quad (1)$$

then the centripetal force acting on it

$$\vec{F} = -m\omega^2 \vec{r} \quad \text{---} \ (2)$$

$$-m\omega^2 \vec{r} = -\frac{GM_e m}{r^3}\vec{r}$$

$$\omega^2 = \frac{GM_e}{r^3} \qquad [\because v = r\omega]$$

$$\frac{v_0^2}{r^2} = \frac{GM_e}{r^3} \Rightarrow v_0 = \sqrt{\frac{GM_e}{r}}$$

This velocity is known as orbital velocity and denote by v_0.

The angular velocity in terms of the period of revolution T, $\omega = \frac{2\pi}{T}$

$$\frac{4\pi^2}{T^2} = \frac{GM_e}{r^3} \Rightarrow T^2 = \frac{4\pi^2}{GM_e} r^3$$

$$\boxed{T^2 \propto r^3}$$

i.e the square of the period of revolution of a satellite moving in a circular orbit round the earth is proportional to the cube of its distance from the centre of the earth.

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Q1 :- The gravitation potential at the centre of solid sphere is how many times that on its surface

(a) $\frac{5}{2}$ (b) $\frac{3}{2}$ (c) $\frac{7}{2}$ (d) $\frac{1}{2}$

Q2 :- The gravitational field inside a thin spherical shell of mass M and radius R at a distance 'r' from its centre is

(a) $-\frac{GM}{r^2}$ (b) $-\frac{GM}{R^2}$ (c) $-\frac{GMr}{R^2}$ (d) zero

Q3 -> Inside a thin uniform spherical shell :.

(a) gravitational field is zero.

(b) gravitational potential is zero.

(c) gravitational field is non-zero but finite

(d) gravitational potential

Q4 :- The relation between g and G is:

(a) $GM_e = gR^2$ (b) $GM_e = g^2R^2$

(c) $GM_e^2 = gR^2$ (d) $G^2M_e^2 = gR^2$

Q5 - The gravitational field outside a solid sphere of mass m at a distance r from the centre.

(a) $-\frac{GM}{r^2}$ (b) zero (c) $-\frac{Gm}{r}$ (d) $-\frac{Gm^2}{r}$