—: UNIT- IV Elasticity :

Elasticity is the property by the virtue of which the material bodies recover (or regain) their original shape and size on the removal of the deforming forces.

Hooke's Law :-

Provided the strain is small, the stress is proportional to the strain.

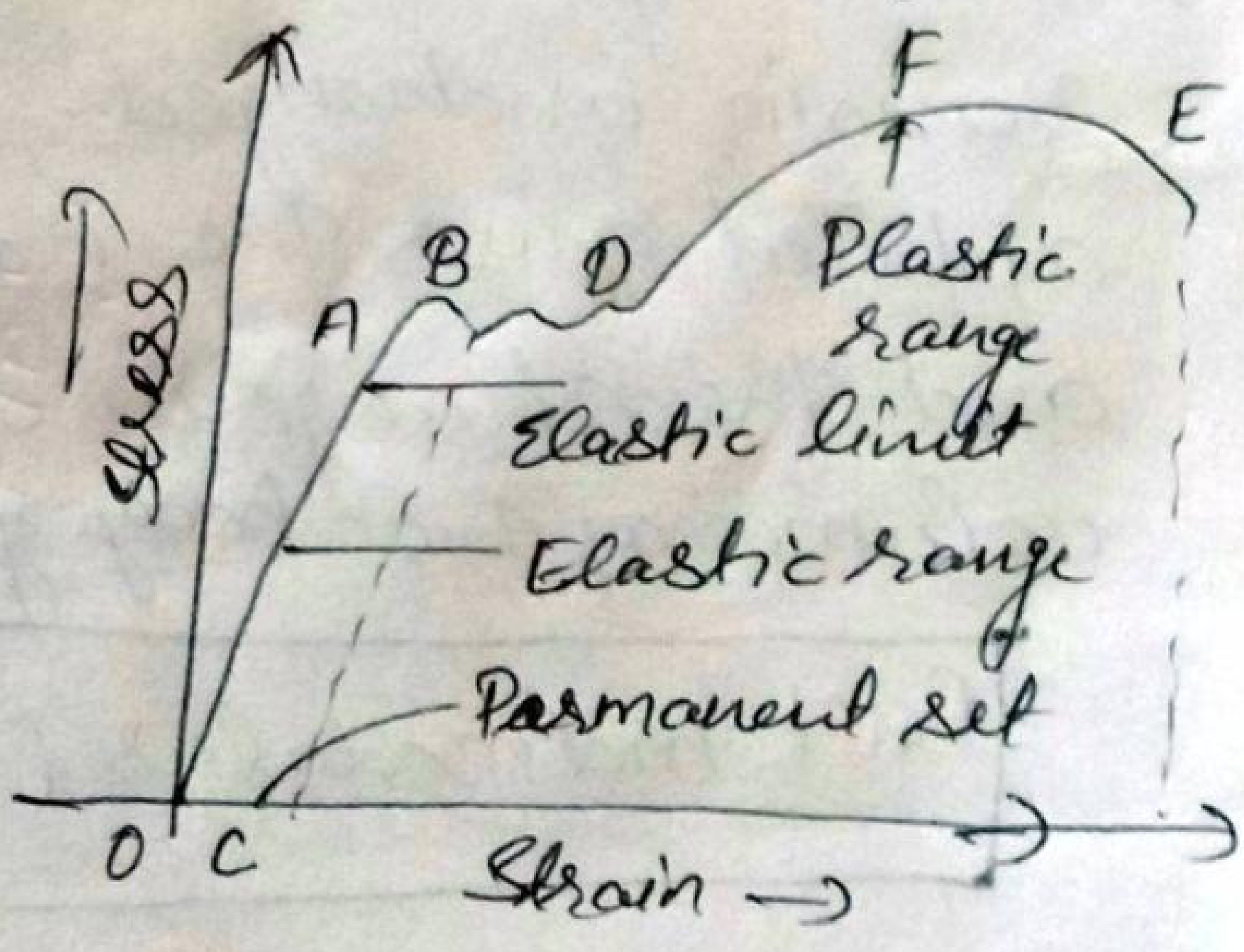
Stress ∝ Strain

Stress = E Strain ⇒ $\boxed{E = \frac{\text{Stress}}{\text{Strain}}}$

Elastic limit :- In the case of a solid, if the stress be gradually increased, the strain too increases with it in accordance with Hooke's law until a point at which the linear relationship between the two just ceases and beyond which the strain increases much more rapidly than is warranted by the law.

Behaviour of a wire or a bar under increasing Stress :-

If we have a wire or bar to gradully increasing stress and plot a graph between the stress applied and the correspoing strain produced we obtain a curve of the form called stres-strain diagram.



Different types of Elasticity:-

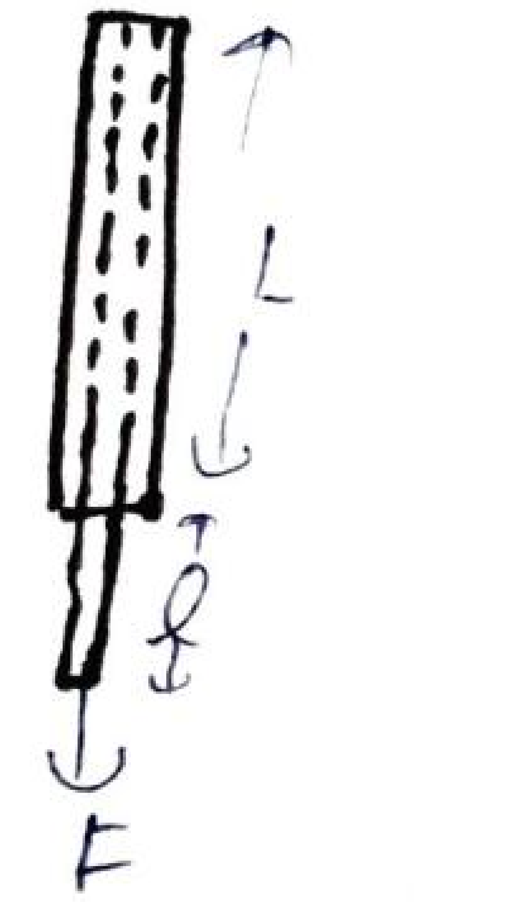
1. Young's modulus (or elasticity of length):-

If F be the force normally to a cross section area A, such that an increase in length l is produced in an original length L, we have

logitudinal stress $= F/A$ and logitudinal strain $= \frac{l}{L}$

$$\boxed{\text{Young's modulus } Y = \frac{F/A}{l/L} = \frac{FL}{lA}}$$

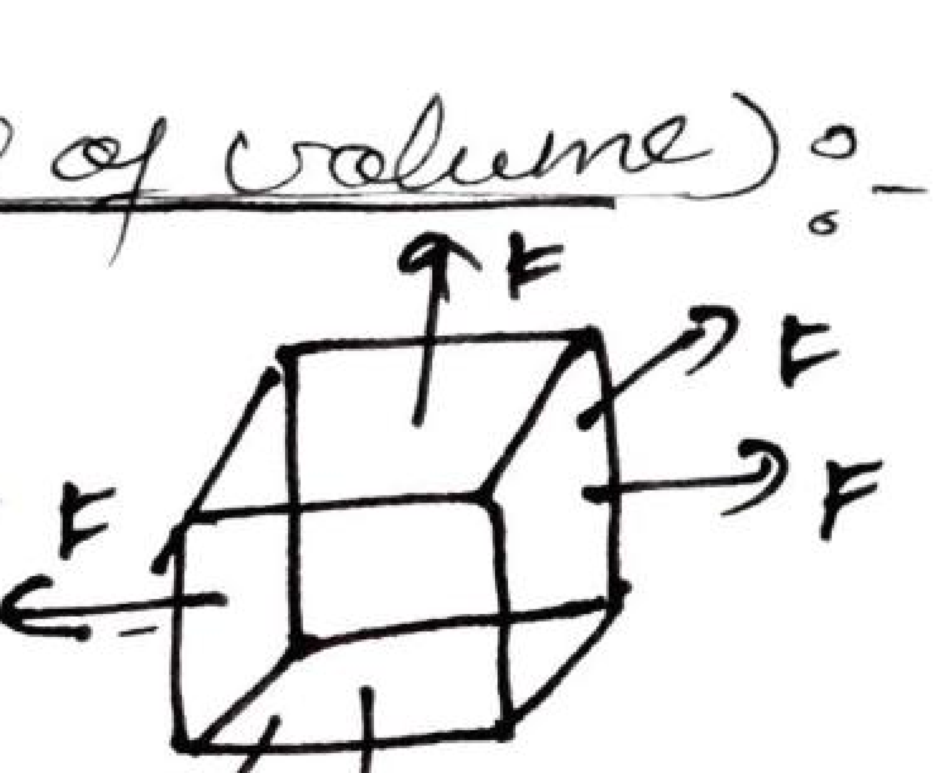
$$= \frac{\text{logitudinal stress}}{\text{logitudinal strain}}$$



Young's modulus for a material is the force required to increase unit length of wire of unit area of cross-section by unity.

2. Bulk modulus (or elasticity of volume):-

When a body is subjected to a uniform pressure perpendicular to its whole surface, it undergoes a change in volume, its shape remain unchanged. The pressure applied gives the normal stress and the change in volume per unit volume of the body gives volume strain.



$$\boxed{\text{Bulk modulus } (K) = \frac{\text{normal stress}}{\text{volume strain}} = -\frac{F/A}{v/V} = -\frac{F}{A}\frac{V}{v} = -\frac{PV}{v}}$$

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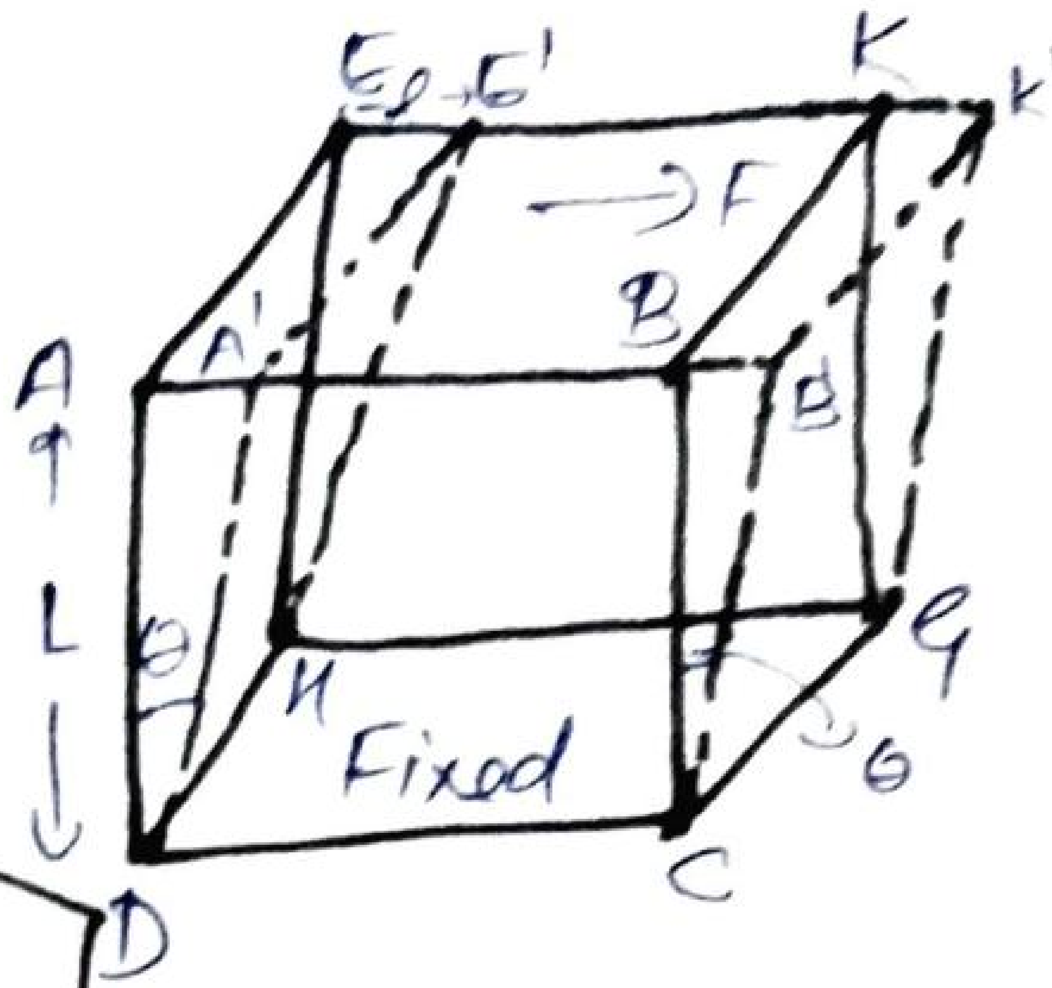
steel and mild

3. Modulus of rigidity (Torsion modulus or Elasticity of shape):-

Modulus of rigidity of a material may be defined as the tangential or shearing stress per unit shear.

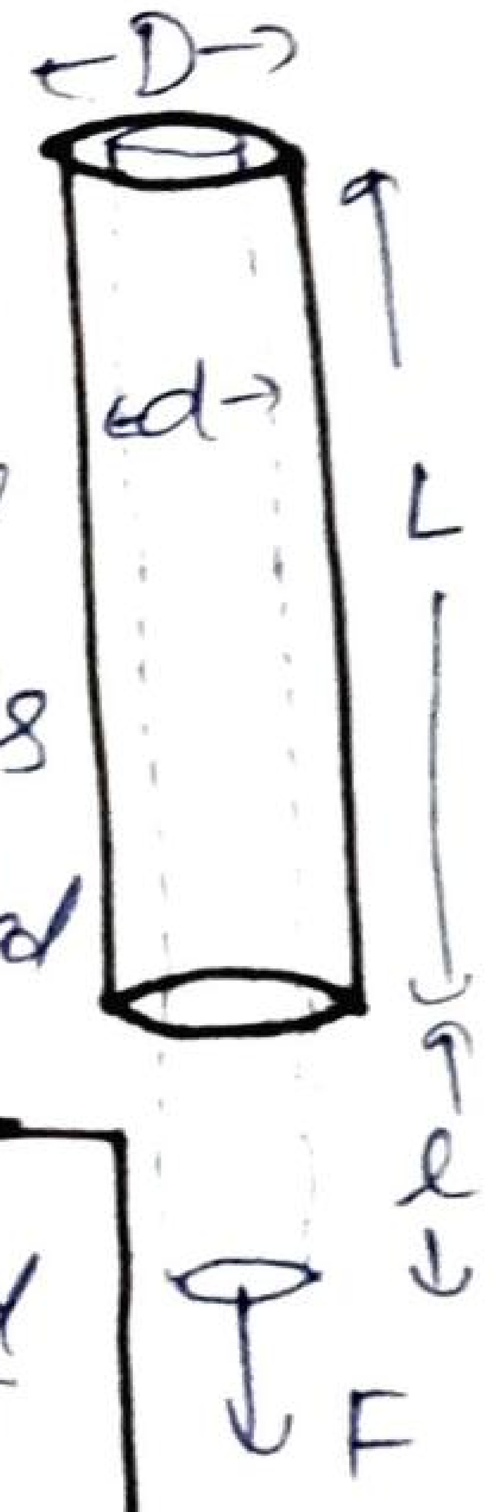


$$\boxed{\eta = \frac{\text{Shearing stress}}{\text{shearing strain}} = \frac{F/A}{\theta} = \frac{F/L^2}{l/L} = \frac{F/A}{l/L}}$$

$$\left[\because \theta = \frac{l}{L}\right]$$

4. Poisson's Ratio:-

Within ~~el~~ elastic limits the ratio of lateral strain to the logitudinal strain is a constant for the material of body and is known as Poisson's ratio ~~after Poisson~~ and is denoted by σ



$$\boxed{\sigma = \frac{\text{Lateral Strain}}{\text{Logitudinal strain}} = -\frac{d/D}{l/L} = -\frac{Ld}{Dl}}$$

Work done:- Logitudinal strain:- $W = \frac{1}{2}$ stress x strain

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Work done in stretching a wire :-

Suppose a wire ~~a~~ is stretched by applying a force (F) ~~as~~ along its length. If L is original length of wire, A cross section area and l is the increase in length. Then we have

$$\text{Young's modulus } Y = \frac{\text{Stress}}{\text{Strain}} = \frac{F/A}{l/L} = \frac{FL}{Al}$$

$$\Rightarrow F = \frac{YAl}{L}$$

Work done during the whole increase in the length of the wire from 0 to l

$$W = \int_0^l F \cdot dl = \int_0^l \left(\frac{YAl}{L}\right) dl = \frac{YA}{L} \frac{l^2}{2}\Big|_0^l$$

$$= \frac{1}{2}\frac{YAl^2}{L} = \frac{1}{2}\frac{(YAl)}{L}l$$

$$\boxed{W = \frac{1}{2} F l} \qquad \left[\because F = \frac{YAl}{L}\right]$$

$$\boxed{W = \frac{1}{2} \times \text{streaching force} \times \text{extension}}$$

Relation between the Elastic constants :-

The elastic constants Y, K, η and σ are all interconnected as will be seen from the following :-

Consider a unit cube ABCDEFGH of unit side and let a force P act normally outwards on each of its six faces.

Thus the forces acting ~~p~~ parallel to x-axis produce extensions along the x-axis and compressions along y and z-axis.

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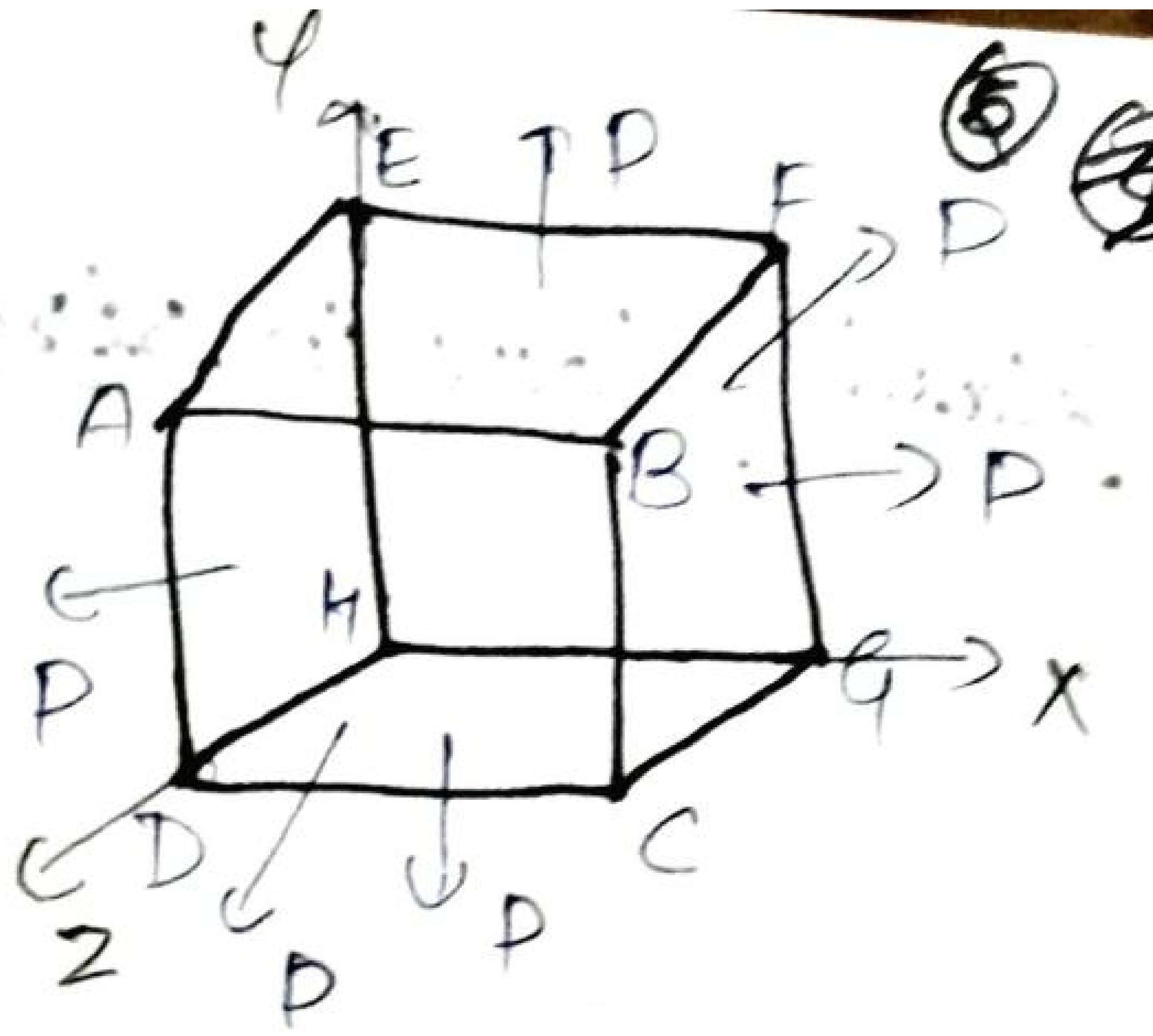
Now Young's modulus

$$Y = \frac{\text{logitudinal stress}}{\text{logitudinal strain}}$$



logitudinal strain

$$= \frac{\text{logitudinal stress}}{Y}$$

$\left[\because \text{stress} = \frac{F}{A} = \frac{P}{1}\right.$

face area is unit

$$\boxed{\text{logitudinal strain} = \frac{P/1}{Y} = \frac{P}{Y}}$$

Extension along x-axis $= \frac{P}{Y}$

~~lateral strain~~

$$\sigma = \frac{\text{lateral strain}}{\text{logitudinal strain}}$$

$$\text{lateral strain} = \sigma \times \text{logitudinal strain}$$

$$= \sigma \times \frac{P}{Y}$$

Contraction along the Y and z-axis $= -\frac{\sigma P}{Y}$

In a similar way considering the pairs of forces acting along Y and z-axes. We have the total extensions e_x, e_y and e_z along X, Y and Z-axes

$$e_x = \frac{P}{Y} - \frac{\sigma P}{Y} - \frac{\sigma P}{Y} = \frac{P}{Y} - \frac{2\sigma P}{Y} = \frac{P}{Y}(1-2\sigma)$$

$$e_y = -\frac{\sigma P}{Y} + \frac{P}{Y} - \frac{\sigma P}{Y} = \frac{P}{Y}(1-2\sigma)$$

$$e_z = -\frac{\sigma P}{Y} - \frac{\sigma P}{Y} + \frac{P}{Y} = \frac{P}{Y}(1-2\sigma)$$

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Now new side of the cube becomes $1+\frac{P}{Y}(1-2\sigma)$.

New volume of the cube be $= \{1+\frac{P}{Y}(1-2\sigma)\}^3$

$$= 1+\frac{3P}{Y}(1-2\sigma)$$

Since the original volume was unity, change in volume

$$= 1+\frac{3P}{Y}(1-2\sigma)-1 = \frac{3P}{Y}(1-2\sigma)$$

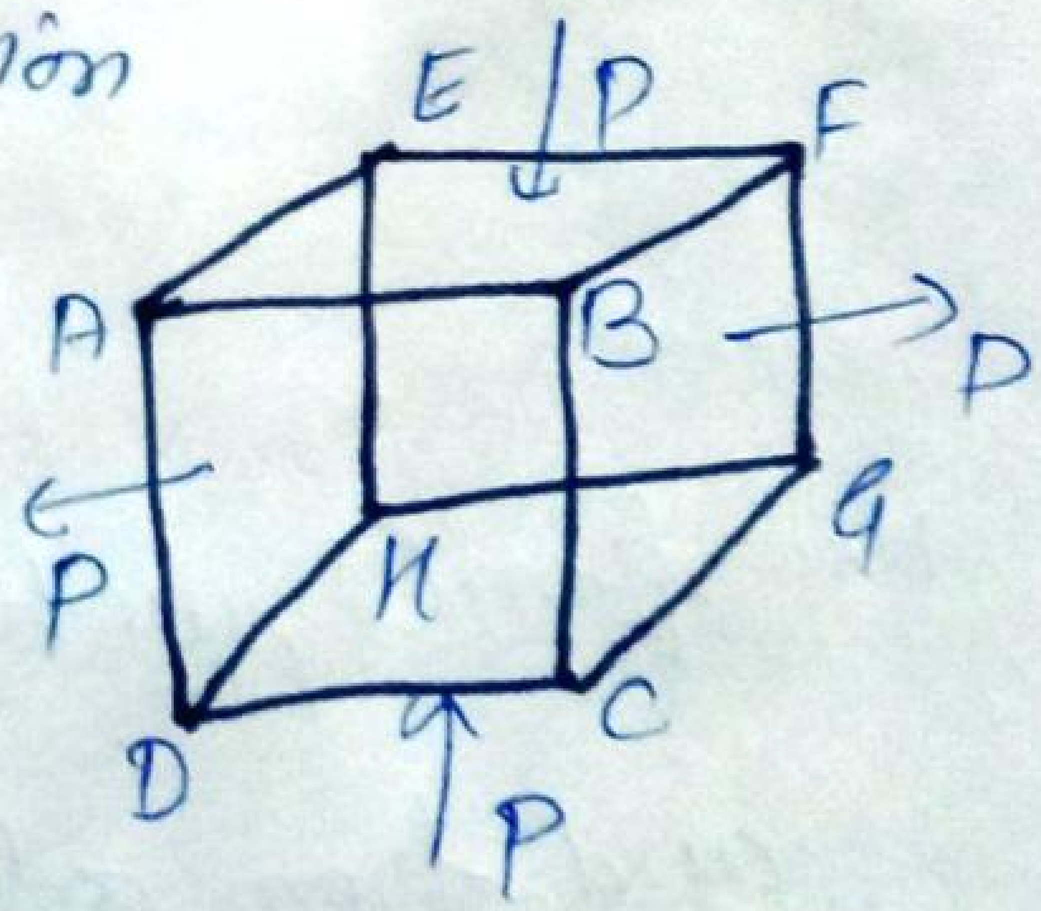
$$\text{volume strain} = \frac{\text{change in volume}}{\text{Original volume}} = \frac{\frac{3P}{Y}(1-2\sigma)}{1}$$

So Bulk modulus K is

$$K = \frac{\text{Volume stress}}{\text{Volume strain}} = \frac{P}{\frac{3P}{Y}(1-2\sigma)}$$

$$\boxed{K = \frac{Y}{3(1-2\sigma)}} \Rightarrow \boxed{Y = 3K(1-2\sigma)} \quad (1)$$

Now instead of extensional stresses on each of the six faces of the cube. Then extensional stress P, parallel to X-axis will produce extension P/Y along x-axis and compression $\frac{\sigma P}{Y}$ along y and z-axes. Similarly compressional stress P parallel to y-axis will produce compression P/Y and extensions $\frac{\sigma P}{Y}$ along each of the X and z-axis.



The net extensions e_x, e_y and e_z along the three axes of X, y and z are, therefore,

$$e_x = \frac{P}{Y} + \frac{\sigma P}{Y} = \frac{P}{Y}(1+\sigma)$$

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$$e_y = -\frac{\sigma P}{Y} - \frac{P}{Y} = -\frac{P}{Y}(1+\sigma)$$

$$e_z = -\frac{\sigma P}{Y} + \frac{\sigma P}{Y} = 0$$

Thus we have equal extension and compression along X and Y axes. But we know that the sum of simultaneous equal compression and extension at right angles to each other are equivalent to a shear θ.

$$\frac{P}{Y}(1+\sigma) + \frac{P}{Y}(1+\sigma) = \theta$$

$$\frac{2P}{Y}(1+\sigma) = \theta$$

$$\boxed{\frac{P}{\theta} = \frac{Y}{2(1+\sigma)}}$$

Therefore, the modulus of rigidity

$$\eta = \frac{\text{shearing stress}}{\text{shear}}$$

$$\eta = \frac{P}{\theta} = \frac{Y}{2(1+\sigma)}$$

$$\boxed{Y = 2\eta(1+\sigma)} \quad \text{——— (2)}$$

Now from (1), we have $\frac{Y}{3K} = 1 - 2\sigma$

and from eq^n (2) $\frac{Y}{\eta} = 2 + 2\sigma$

adding these $\frac{Y}{3K} + \frac{Y}{\eta} = 3 \Rightarrow \boxed{Y = \frac{3\eta K}{3K+\eta}}$ ——— (3)

From eq^n (1) and eq^n (2)

$$3K(1-2\sigma) = 2\eta(1+\sigma)$$

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$$3K-2\eta=\sigma(2\eta+6K)$$

$$\boxed{\sigma=\frac{3K-2\eta}{6K+2\eta}} \quad \text{--- (4)}$$

Eq^n (1), (2), (3) and (4) are the relations connecting the four elastic constants.

<u>Limiting values of σ :-</u>

from eq^n (1) and eq^n (2)

$$3K(1-2\sigma)=2\eta(1+\sigma)$$

If σ is positive, the expression on the right hand side in the relation will be positive.

$$1-2\sigma>0$$

$$1>2\sigma$$

$$\sigma<\frac{1}{2} \quad \text{or} \quad \boxed{\sigma<0.5}$$

If σ is negative, left hand side is positive

$$1+\sigma>0 \Rightarrow \boxed{\sigma>-1}$$

Thus, theoretically, the limiting values of σ are -1 and 0.5, though in actual practice it lies between 0.2 and 0.4 for most of the materials.

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Unit - IV MCQS

1. The potential energy per unit volume of a stretched wire is

(a) $\frac{1}{2}$ x load x extension
(b) $\frac{1}{2}$ x stress x strain
(c) stress x strain
(d) load x extension

(2) The potential energy of a stretched wire is

(a) $\frac{1}{2}$ x load x extension
(b) $\frac{1}{2}$ x stress x strain
(c) stress x strain
(d) load x extension

(3) The relation between elastic constants Y, K and σ is

(a) $Y = 2K(1-2\sigma)$
(b) $Y = 3K(1-2\sigma)$
(c) $Y = 3K(1-3\sigma)$
(d) $Y = 2K(1-\sigma)$

(4) The relation between Y, K and η is

(a) $\frac{3}{Y} = \frac{3}{K} + \frac{1}{\eta}$
(b) $\frac{1}{Y} = \frac{1}{9K} + \frac{1}{3\eta}$
(c) $\frac{9}{Y} = \frac{1}{K} + \frac{3}{\eta}$
(d) $\frac{1}{\eta} = \frac{1}{K} + \frac{1}{Y}$

(5) The values of Possion's ratio (σ) lies ~~but~~ between

(a) 0.5 and -1
(b) -0.5 and +1
(c) -0.5 and -1
(d) -0.5 and +0.5

(6) The change is shape of a regular body is due to

(a) Bulk strain
(b) shearing strain
(c) longitudinal strain
(d) metallic strain

(7) The correct relation is

(a) $Y > \eta$
(b) $\sigma < -1$
(c) $\sigma = \frac{Y}{2\eta} - 1$
(d) $\sigma = \frac{3K}{Y}$

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8. The torsional rigidity for solid cylinder is (restoring couple per unit twist):
   (a) $C = \frac{\pi \eta r^4}{2l}$ (b) $C = \frac{\pi \eta r^4}{4l}$
   (c) $C = \frac{\pi \eta r^2}{2l}$ (d) $C = \frac{\pi^2 \eta r^4}{4l}$

9. Which of the following is correct?
   (a) $\sigma = \frac{3K+2\eta}{2\eta+6K}$ (b) $\frac{3K-2\eta}{6K+2\eta}$
   (c) $\sigma = \frac{3K+2\eta}{2\eta-6K}$ (d) $\frac{3K-2\eta}{2\eta-6K}$

10. Young's modulus of a perfectly plastic body is
    (a) zero (b) infinite
    (c) 1 (d) none

11. If Y_S and Y_R are the Young's modulus of steel and rubber respectively, then
    (a) $Y_S > Y_R$ (b) $Y_S < Y_R$
    (c) $Y_S = Y_R$ (d) none

12. Torsional rigidity of a rod is directly proportional to
    (a) radius^2 (b) radius^3 (c) radius^4 (d) radius^5

13. A metallic wire of length L meters extends by l meters when stretched by suspending a weight Mg to it. The mechanical energy stored in the wire is
    (a) 2Mgl (b) $\frac{1}{2}$Mgl (C) Mgl (d) $\frac{1}{4}$Mgl [∵ F=m

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